

4.1 Review and Preview

"Probability is the underlying foundation on which the important methods of inferential statistics are built."

→ Statisticians reject explanations based on very low probabilities (the Rare Event Rule for Inferential Statistics)

4.2 Basic Concepts of Probability

Probability (P):

→ expressed as numbers between 0 and 1 (where 0 is an impossible event & 1 is a certain event)

Ex: $P(\text{Thanksgiving falls on a Wednesday})$
 $P(\text{Thanksgiving falls on a Thursday})$

→ must learn how to interpret probability values

→ Example procedures: rolling a die, answering multiple-choice questions, ...

Rounding off probabilities: either give the exact fraction or round off final decimal results to 3 significant digits (all digits except zeros that are included for proper placement of the decimal point)

→ Ex: $P = 0.021491$ rounds to

$P = 1/3$ rounds to

$P = 432/7842$ rounds to

$P(\text{heads}) =$

Event:

→ Notation: A , B , and C denote specific events

Simple event:

Sample space:

→ Ex: A woman wants to have 3 babies—what are the possible gender combinations?

- simple event:
- not a simple event:
- sample space:

Probability of event A occurring: $P(A)$ found using 3 approaches

1. Relative Frequency Probability:

$P(A) =$

- You may remember this as “Empirical Probability”
- Ex #1: $P(\text{tack lands point up})$: toss the tack many times & then find the ratio of the number of times the tack lands point up to the total number of tosses
- Ex #2: A recent survey of 1010 adults in the US showed that 202 of them smoke. Find the probability that a randomly selected adult in the US is a smoker
- Ex #3: Your teacher recorded that you were absent from your statistics class 4 times out of the last 38 days. Find the probability that you will be absent on any given day.

- Law of large numbers:
- Computer simulations are often used to repeat the procedure many times

2. Classical Probability:

$P(A) =$

- You may remember this as "Theoretical Probability"
- calculations can become very complex & the probability of each simple event must have an equal chance of occurring
- Ex #1: $P(2)$ with a balanced & fair die
- Ex #2: Find the probability that when a couple has 3 children, they will have exactly 2 boys.
- Ex #3: Find the probability that if you flip a coin 3 times, you will get tails more than once.

3. Subjective Probability:

- Ex #1: Meteorologists use their expert knowledge of weather conditions to determine the probability of rain tomorrow
- Ex #2: What is the probability that you will get stuck in the next elevator that you ride?

Probabilities & Outcomes that are NOT EQUALLY LIKELY:

Ex: the probability that a Republican will win the next election is not $\frac{1}{2}$

Complement of an event (\bar{A}):

- Ex #1: In reality, more boys are born than girls. In one typical group, there were 205 newborn babies and 105 of them were boys. If one baby is randomly selected, what's the probability that the baby is NOT a boy?

- Ex #2: You are choosing marbles out of a bag that contains 6 red, 4 yellow, and 5 green marbles. If one marble is randomly chosen, what is the probability that you do not choose yellow?

Unlikely and Unusual Events:

- Unlikely events: small probability (0.05 or less)
- Unusual events: Extreme result (number of outcomes of a particular type is far below or far above the typical values)
- Ex #1: When a fair coin is tossed 1000 times, the result consists of exactly 500 heads. The probability of getting exactly 500 heads in 1000 tosses is 0.0252. Is this result unlikely? Is 500 heads unusually low or unusually high?
- Ex #2: When a fair die is rolled 100 times, you roll a six 24 times. The probability of rolling a six exactly 24 times out of 100 rolls is 0.016. Is this result unlikely? Is rolling a six 24 times unusually low or unusually high?

Rare Event Rule:

Ex: In a clinical experiment of the Salk vaccine for polio, 200,745 children were given the placebo and 201,229 were given the vaccine. There were 115 cases of polio among those in the placebo group and 33 cases of polio in the treatment group. If we assume that the vaccine has no effect, the probability of getting such test results is found to be less than 0.001. What does that probability imply about the effectiveness of the vaccine?

Odds:

- Ex: 50:1 or "50 to 1"
- Disadvantage: make calculations extremely difficult for statisticians, mathematicians, & scientists
- Advantage: make it easier to deal with money transfers associated with gambling (used in casinos, lotteries, & racetracks)
- Actual odds against event A occurring:
 - usually expressed in the form $a:b$ or "a to b" (where a and b are integers having no common factors)
- Actual odds in favor of event A:

- Payoff odds against event A:

payoff odds against event A =

- Ex #1: If you bet \$5 on #13 in roulette, your probability of winning is $1/38$ and the payoff odds are given by the casino as 35:1

--Find the actual odds against the outcome of 13:

--Find the odds in favor of the outcome of 13:

--How much net profit would you make if you win by betting on 13?

--If the casino was operating just for fun & the payoff odds were changed to match the actual odds against 13, how much would you win if the outcome was 13?

→ Ex #2: You are playing the Texas Pick 3 lottery. You can bet \$1 by selecting the exact order of three digits between 0 and 9. The probability of winning is $1/1000$. If the same three numbers are drawn in the same order, you collect \$500, so your net profit is \$499.

--Find the actual odds against winning:

--Find the payoff odds if you bet \$2

--If a website states that the odds for the Texas Pick 3 Lottery is 1:1000, is that description accurate?

4.3 Addition Rule

Compound event:

Addition Rule:

- Finds the total number of ways without counting any outcomes more than once
- "or" ("inclusive or"): either one or the other or both
- Also written as $P(A \cup B)$

<u>Pre-Employment Drug Screening</u>		
	Subject Uses Drugs	Subject Does Not Use Drugs
Positive Test Result	119 (true positive)	24 (false positive)
Negative Test Result	3 (false negative)	154 (true negative)

→ Ex #1: Assume that 1 person is randomly selected from the 300 employees that were tested for drug use. Find the probability of selecting a subject who had a positive test result or used drugs.

→ Ex #2: Assume that 1 person is randomly selected from the 300 employees that were tested for drug use. Find the probability of selecting a subject who had a negative test result or does not use drugs.

Formal Addition Rule:

- Subtracts the outcomes that are counted twice
- $P(A \text{ and } B)$ is also written as $P(A \cap B)$

<u>Pre-Employment Drug Screening</u>		
	Subject Uses Drugs	Subject Does Not Use Drugs
Positive Test Result	119 (true positive)	24 (false positive)
Negative Test Result	3 (false negative)	154 (true negative)

→ Ex #1: $P(\text{positive test result OR used drugs}) =$

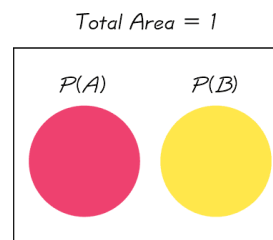
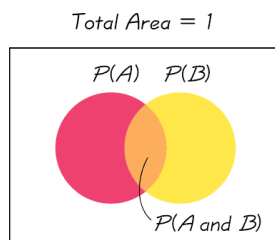
→ Ex #2: $P(\text{negative test result OR does not use drugs}) =$

Events A and B are Mutually Exclusive (or disjoint):

→ Ex #1: Determine whether the following events are mutually exclusive:

A: getting a subject with a negative test result

B: getting a subject who didn't use drugs



→ Ex #2: Determine whether the following events are mutually exclusive for a single trial:

Passing your first statistics test & Failing your first statistics test

Complementary Events:

$$\rightarrow P(A \text{ or } \bar{A}) = P(A) + P(\bar{A}) = 1$$

$$\rightarrow P(\bar{A}) = 1 - P(A)$$

$$\rightarrow P(A) = 1 - P(\bar{A})$$

→ The use of the rule of complementary events can greatly simplify certain problems

→ Ex #1: In reality, when a baby is born, $P(\text{boy}) = 0.512$. Find $P(\overline{\text{boy}})$.

→ Ex #2: When the author observed a sobriety checkpoint conducted by the Dutchess County Sheriff Department, he saw that 676 drivers were screened and 6 were arrested for driving while intoxicated. Based on those results,

- What estimate can we use for $P(I)$ where I denotes the event of screening a driver and getting someone who is intoxicated?
- What does $P(\bar{I})$ represent?
- What is the value of $P(\bar{I})$?

4.4 Multiplication Rule: Basics

Basic Multiplication Rule:

- Must adjust the probability if the outcome of the 1st event A somehow affects the outcome of the 2nd event B
- Also written as $P(A \cap B)$
- Ex: You go out to eat at a restaurant and they have the following menu:
 - Appetizers: soup, salad, & stuffed clams
 - Entrees: pasta, chicken, beef, & shrimp
 - Desserts: mousse & cheesecake
 - Create a tree diagram of all possible 3 course meals:
- How many 3 course meals are possible?
- How else could you have found the number of possibilities?
- This only works if the events are independent of one another

Conditional Probability: $P(B|A)$

- read as "B given A"
- 2 events are independent
- Sampling with replacement: selections are independent events
- Ex: playing the California lottery & then playing the New York lottery

→ 2 events are dependent

-Sampling without replacement: selections are dependent events

-Ex: the event of having your car start and the event of getting to your statistics class on time

Formal Multiplication Rule:

→ Independent event example: Find the probability of tossing a coin 3 times & getting all heads.

→ Dependent event example: Find the probability of choosing 2 hearts in a deck of 52 playing cards (without replacing the first card before choosing the second).

→ Ex #1: In the 108th Congress, the Senate consists of 51 Republicans, 48 Democrats & 1 Independent. If a lobbyist randomly selects three senators, what is the probability they are all Republican?

→ Ex #2: Airport baggage scales can show that bags are overweight and high additional fees can be imposed. The NYC Department of Consumer Affairs checked all 810 scales at JFK and LaGuardia airports, and 102 scales were found to be defective.

--If 2 of the 810 scales are randomly selected *with replacement*, find the probability that they are both defective.

--If 2 of the 810 scales are randomly selected *without replacement*, find the probability that they are both defective.

Treating Dependent Events as Independent: "If a sample size is no more than 5% of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent)."

→ Guideline used by pollsters when surveying 1000 adults from a population of millions.

4.5 Multiplication Rule: Complements & Conditional Probability

Complements: The Probability of "At Least One"

- "at least one" is equivalent to "one or more"
- Complement: get no items of that type
- $P(\text{at least one}) =$
- Ex #1: a couple plans to have 3 children & want to know the probability of having at least 1 girl

- Ex #2: you flip a coin twice and want to know the probability of getting at least 1 tail

Conditional Probability:

$$P(B|A) =$$

- Confusion of the Inverse: incorrectly believe that $P(B|A)$ and $P(A|B)$ are the same or to incorrectly use one value for the other

Ex: Consider the probability that it is dark outside, given that it is midnight. $P(\text{dark}|\text{midnight}) = 1$ (conveniently ignoring the Alaskan winter and such locations). But the probability that it is midnight, given that it is dark outside is almost zero. Therefore, $P(\text{dark}|\text{midnight}) \neq P(\text{midnight}|\text{dark})$

<u>Pre-Employment Drug Screening</u>		
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Positive Test Result	119 (true positive)	24 (false positive)
Negative Test Result	3 (false negative)	154 (true negative)

→ Ex #1: If 1 of the 300 subjects is randomly selected, find the probability that the person tested positive, given that he or she actually used drugs.

→ Ex #2: If 1 of the 300 subjects is randomly selected, find the probability that the person actually used drugs, given that he or she tested positive.

4.6 Counting

Fundamental Counting Rule:

- Ex #1: To win a given lottery, you must choose 4 digits between 0 and 9 and repetitions are allowed. How many different lottery numbers are possible?
- Ex #2: Assume a criminal is found using your social security number and claims all the digits were randomly generated. What's the probability of getting your social security number when randomly generating 9 digits?
- Ex #3: You are in charge of planning an evacuation plan out of your office building. If there are two ways to exit your office on the 2nd floor and five ways to exit the building from the 1st floor, how many different evacuation routes are possible?

Factorial Rule:

- Factorial (!) denotes the product of decreasing positive whole numbers
- Ex: $4! =$
- Zero Factorial: $0! =$
- Ex #1: You're planning a trip to Disney & you want to get through the following 5 rides on the 1st day: Space Mountain, Tower of Terror, Rock 'n' Roller Coaster, Mission Space, and Dinosaur. How many different routes are possible?

→ Ex #2: You are in charge of organizing the science projects for this week's science fair. If there are 12 different projects, in how many ways could the projects be arranged?

→ Ex #3: You deliver newspapers as your part-time job. If you have 8 houses to deliver the paper to before school, how many different routes are possible?

***routing problems are often referred to as "the traveling salesman problem" because they are important to so many different companies & the number of routes can be very large*

→ *Calculator:*

- *Type the number before the factorial symbol*
- *Press Math → Prb → 4:!*

Permutations Rule (when items are all different):

→ There are n different items available

→ We select r of the n items (without replacement)

→ Permutations, arrangements, & sequences all consider rearrangements of the same items to be different

→ Ex #1: We must conduct surveys in state capitals but only have time to visit 4 capitals. How many possible routes are there?

→ Ex #2: We must hand out fundraiser information to the 10 senior homerooms but only have 4 homeroom representatives available today. How many possible routes are possible?

→ *Calculator:*

- *Type the n value first*
- *Press Math → Prb → 2:nPr*

Permutations Rule (when some items are identical to others):

- There are n items available and some items are identical to others
- We select all n of the items (without replacement)
- We consider rearrangements of distinct items to be different sequences

Number of permutations =

- Ex #1: How many ways can the letters of the word MISSISSIPPI be arranged?

- Ex #2: How many ways can the letters of the word TRIGONOMETRY be arranged?

*When different orderings of the same items are counted separately, we have a permutation problem. When different orderings of the same items are NOT counted separately, we have a combination problem.

Combinations Rule:

- We select r of the n different items (without replacement)
- We consider rearrangements of the same items to be the same
- number of combinations of r items selected from n different items:
$${}_nC_r =$$
- Ex #1: In the Pennsylvania Match 6 lottery, winning the jackpot requires that you select six different numbers from 1 to 49, and the same six different numbers must be drawn in the lottery. If the winning numbers can be drawn in any order, find the probability of winning the jackpot when one ticket is purchased.
- Ex #2: In the Texas Pick 4 lottery, winning the jackpot requires that you select four different numbers from 0 to 49, and the same four different numbers must be drawn in the lottery. If the winning numbers can be drawn in any order, find the probability of winning the jackpot when one ticket is purchased.
- Calculator:
 - Type the n value first
 - Press Math → Prb → 3: ${}_nC_r$

Permutation vs. Combination Example: When testing a new drug on humans, Phase I of a clinical test is conducted with a relatively small number of healthy volunteers. Let's assume we want to treat 8 healthy humans with a new drug, and we have 10 suitable volunteers available.

- If the subjects are selected & treated in sequence, so that the trial is discontinued if anyone presents an adverse reaction, how many different sequential arrangements are possible if 8 people are selected from the 10 that are available?

- If 8 subjects are selected from the 10 that are available, & the 8 subjects are all treated at the same time, how many different treatment groups are possible?

4.7 Probabilities Through Simulations

Simulation of a procedure:

- Using a table of random digits, or a calculator / computer to randomly generate numbers in such a way that they are equally likely

- You can use a TI graphing calculator to randomly generate numbers (see the screen to the right)

```
randInt(1,365,25  
→L1  
(79 206 340 133...  
SortA(L1)           Done  
L1  
(17 34 46 70 79...
```